Name: Date: Period: 1

Perpendicular and Angle Bisectors

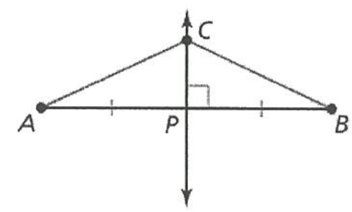
son Objective IWBAT prove and apply thms about $\perp$ bisectors and $\angle$ bisectors

In Chapter 3, you learned that a *perpendicular bisector* of a line segment if the line that is ⊥ to the segment at its midpoint.

Equidistant: A point is equidistant from two figures when the point is the same distance from each figure.

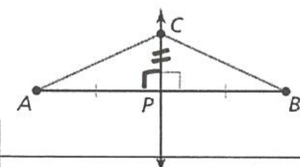
Perpendicular Bisector Theorem

In a plane, if a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.



PROOF: **Given**: $\overleftrightarrow{CP}$ is the ⊥ bisector of $\overline{AB}$

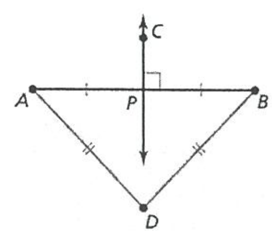
Prove: CA = CB



Statements	Reasons
① $\overleftrightarrow{CP}$ is $\perp$ bisector of $\overline{AB}$	① given
② $\overline{CP} \cong \overline{CP}$	② reflex. prop $\cong$
③ $\overline{AP} \cong \overline{BP}$, $\angle APC, \angle BPC$ are right $\angle$s	③ def. of $\perp$ bisector
④ $\angle APC \cong \angle BPC$	④ all right $\angle$s are $\cong$
⑤ $\triangle APC \cong \triangle BPC$	⑤ SAS $\cong$
⑥ $\overline{CA} \cong \overline{CB}$	⑥ CPCTC
⑦ CA = CB	⑦ $\cong \rightarrow =$

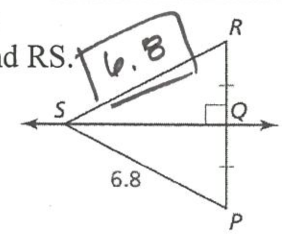
Converse of the Perpendicular Bisector Theorem

In a plane, if a point is equidistant from the endpoints of a segment, then it lies on the $\perp$ bisector of the segment.

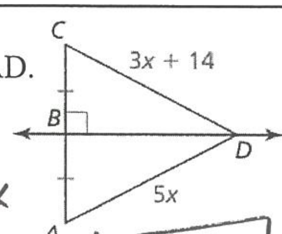


Examples:

1. Find RS. $\boxed{6.8}$



2. Find AD.

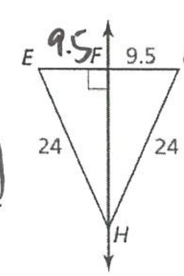


$3x+14=5x$

$x=7$

$\boxed{AD=35}$

3. Find EG

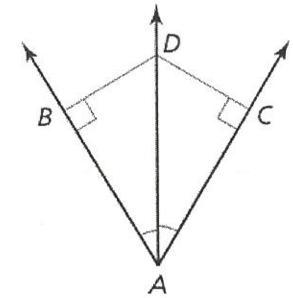


$\boxed{19}$

In Chapter 1, you learned that an *angle bisector* is a ray that divides an angle into two congruent adjacent angles.

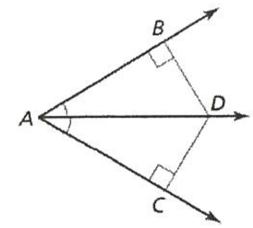
You also learned that the *distance from a point to a line* is the length of the ⊥ segment from the point to the line.

So, in the figure at right, $\overrightarrow{AD}$ is the bisector of $\angle BAC$, and the distance from point D to $\overrightarrow{AB}$ is DB, where $\overline{DB} \perp \overrightarrow{AB}$



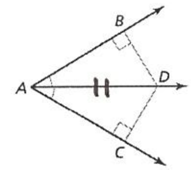
Angle Bisector Theorem

If a point lies on the bisector of an angle, then it is equidistant from the two sides of the angle.



PROOF: **Given**: $\overrightarrow{AD}$ bisects $\angle BAC$, $\overline{DB} \perp \overrightarrow{AB}$, $\overline{DC} \perp \overrightarrow{AC}$

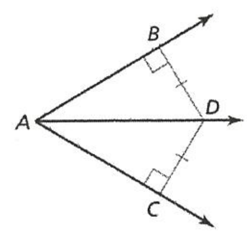
Prove: DB = DC



Statements	Reasons
① $\overrightarrow{AD}$ bisects $\angle BAC$, $\overline{DB} \perp \overrightarrow{AB}$, $\overline{DC} \perp \overrightarrow{AC}$	① given
② $\angle BAD \cong \angle CAD$	② def. of ∠ bisector
③ $\angle DBA$, $\angle DCA$ are right ∠s	③ def. of ⊥ lines
④ $\angle DBA \cong \angle DCA$	④ all right ∠s are ≅
⑤ $\overline{AD} \cong \overline{AD}$	⑤ reflex. prop. ≅
⑥ $\triangle ABD \cong \triangle ACD$	⑥ AAS ≅
⑦ $\overline{DB} \cong \overline{DC}$ ⑧ DB = DC	⑦ CPCTC ⑧ ≅ → =

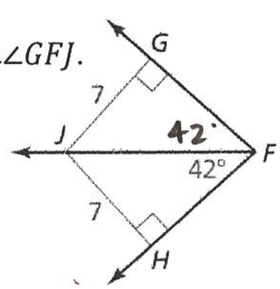
Converse of the Angle Bisector Theorem

If a point is in the inside of an angle and is equidistant from the two sides of the angle, then it lies on the ∠ bisector of the angle.

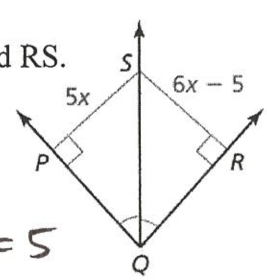


Examples:

4. Find $m\angle GFJ$.



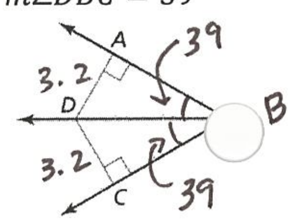
5. Find RS.



x = 5

$\boxed{RS = 25}$

6. Find $m\angle ABC$ when AD = 3.2, CD = 3.2 and $m\angle DBC = 39°$



$\boxed{78°}$

Writing Equations of Perpendicular Bisectors

Example:

7. Write an equation of the perpendicular bisector of the segment with endpoints P(-2, 3) and Q(4, 1)

Step 1: Find the midpoint of the original segment. Why? bisector: intersects @ mdpt

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+4}{2}, \frac{3+1}{2}\right)$$

$$\left(\frac{2}{2}, \frac{4}{2}\right) \quad (1, 2)$$

Step 2: Find the slope of the original segment. Why? need the ⊥ slope...

$$\frac{y_1-y_2}{x_1-x_2} = \frac{3-1}{-2-4} = \frac{2}{-6} = -\frac{1}{3}$$

Step 3: Find the slope of the segment *perpendicular* to the original segment. How? slopes must multiply to -1

$$m_\perp = 3$$

$$\left(-\frac{1}{3}\right)(3) = -1$$

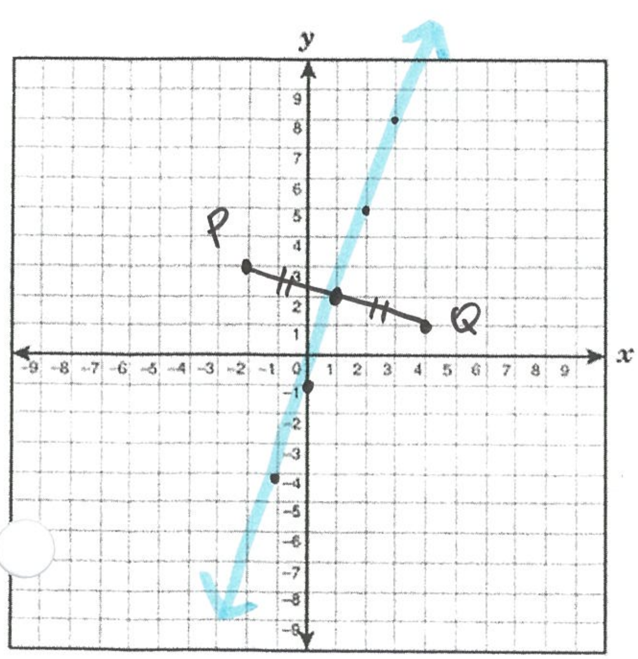
Step 4: Write the equation.

Use point-slope form ($y-y_1=m(x-x_1)$) OR slope-intercept form ($y=mx+b$)

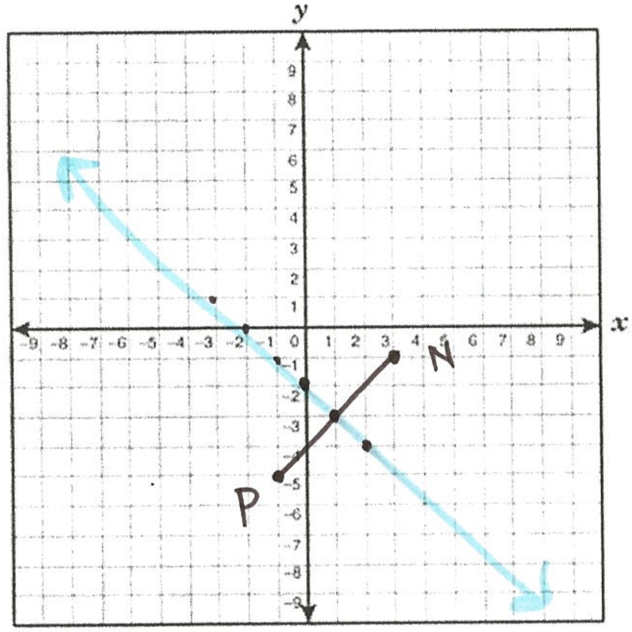
$$y-2 = 3(x-1)$$

$$y-2 = 3x-3$$

$$\boxed{y = 3x-1}$$



8. Write the equation of the perpendicular bisector of the segment with endpoints P(-1, -5) and N(3, -1)



① mdpt : $(1, -3)$

② slope : $m = 1$

③ $\perp$ slope : $m_\perp = -1$

④ eqn: $y - (-3) = -1(x - 1)$

$y + 3 = -x + 1$

$\boxed{y = -x - 2}$

Not nice #s example

Find the eqn of the $\perp$ bisector

w/ endpoints $Y(10,-7)$ and $Z(-4,1)$

$$\text{mdpt} = \left(\frac{10+(-4)}{2}, \frac{-7+1}{2}\right)$$

$$= \left(\frac{6}{2}, -\frac{6}{2}\right)$$

$$= (3,-3)$$

$$\text{orig. slope} = \frac{-7-1}{10-(-4)} = \frac{-8}{14} = -\frac{4}{7}$$

$$\perp \text{ slope} = \frac{7}{4}$$

$$y-(-3) = \frac{7}{4}(x-3)$$

$$y+3 = \frac{7}{4}x - \frac{21}{4}$$

$$y = \frac{7}{4}x - \frac{21}{4} - 3$$

$$y = \frac{7}{4}x - \frac{21}{4} - \frac{12}{4}$$

$$\boxed{y = \frac{7}{4}x - \frac{33}{4}}$$